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## THE STUDY OF THE STRENGTH OF DENTURES WITH DIFFERENT SURFACE RELIEFS UNDER THE ACTION OF STATIC LOADING

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**Key words:** dentures, boundary element method, stress-strain state, statistical loads

**Ключові слова:** зубні протези, метод граничних елементів, напружено-деформований стан, статичне навантаження

**Ключевые слова:** зубные протезы, метод граничных элементов, напряженно-деформованное состояние, статистические нагрузки

**Abstract. The study of the strength of dentures with different surface reliefs under the action of static loading.**

**Kolomiets L.V., Orobey V.F., Lymarenko O.M., Ovcharov Y.V., Tsilvik O.V.** *The paper presents modeling of denture specimens with different surface reliefs under the action of static loading using modern numerical methods - finite and boundary elements. Based on the analysis of literature, it is shown that the most effective approaches for calculating the stress-strain state (SSS) of dentures is the finite element method (FEM), which allows you to create 3D models with any complexity of geometry and surface, as well as the boundary element method (BEM), which allows in some cases to obtain more accurate calculation results than finite element method. The algorithm and procedure for exact integration of differential equations of hollow shells according to the algorithm of Kantorovich-Vlasov variational method are presented for the formation of the calculated finite element method ratios. The analytical expressions for the parameters of the hollow shells used for calculations of the state of dentures are given. Finite element method is represented in the work by the universal package SolidWorks. Different stages of solid-state modeling of prototypes of dentures with different surface reliefs are shown in detail. Dental stress-strain state calculations were performed by two methods. The results of the calculations are in good agreement with each other, which proves the reliability of both models developed and the results of the stress-strain state received. It is shown that the smallest values of stresses occur in dentures with a rhombic lattice; they are 5.9% less than prosthesis with a smooth surface, and 18.78% less than in prosthesis with a square lattice. Equivalent displacement of a rhombic lath prosthesis is less by 3.864% than that of a smooth surface prosthesis and 8 52% less than a square lattice prosthesis.*

**Реферат. Дослідження міцності зубних протезів з різними поверхневими рельєфами при дії статичного навантаження. Коломієць Л.В., Оробей В.Ф., Лимаренко О.М., Овчаров Ю.В., Цільвік О.В.** *У роботі представлено моделювання зразків зубних протезів з різними поверхневими рельєфами при дії статичного навантаження за допомогою сучасних чисельних методів – скінченних та граничних елементів. На підставі проведеного аналізу літературних джерел показано, що найбільш ефективними підходами для розрахунку напружено-деформованого стану (НДС) зубних протезів є метод скінченних елементів (МСЕ), який дозволяє створювати 3D моделі з будь-якою складністю геометрії і поверхні, а також метод граничних елементів (МГЕ), який дозволяє в ряді випадків отримувати більш точні результати розрахунків порівняно з МСЕ. Для формування розрахункових співвідношень МГЕ представлені алгоритм і процедура точного інтегрування диференціальних рівнянь пологих оболонок за алгоритмом варіаційного методу Канторовича-Власова. Наведені аналітичні вирази для параметрів пологих оболонок, що використовуються для розрахунків стану зубних протезів. МСЕ в роботі представлений універсальним пакетом SolidWorks. Детально показані різні етапи твердотілого моделювання дослідних зразків зубних протезів з різними поверхневими рельєфами. Виконано розрахунки НДС зубних протезів двома методами. Результати розрахунків добре узгоджуються*

між собою, що доводить достовірність як розроблених моделей, так і результатів отриманого НДС. Показано, що найменші значення напружень виникають у протезі з ромбічними решітками, вони на 5,9% менше, ніж у протеза з гладкою поверхнею, і на 18,78% менше, ніж в протеза з квадратними решітками. Еквівалентні переміщення в протеза з ромбічними решітками менше на 3,864%, ніж у протеза з гладкою поверхнею, і на 8,52% менше, ніж у протеза з квадратними решітками.

Modern dentistry is impossible without the use of mathematical modeling of processes occurring in a complex dentition of a person.

In works [1, 2, 3], the high-precision models of the three-dimensional problem of elasticity theory to assess the influence of the level of various factors on the qualitative picture of the stress-strain state (SSS) of the dentogingival system were used by the authors.

However, the works analysing SSS of dental orthopedic constructions with the most modern and effective numerical calculation method – the boundary element method (BEM) in various ways are absent. Therefore, research aimed at using BEM for calculating elements of orthopedic structures with complex geometry, with disorders of the physical and mechanical properties of materials and complex boundary conditions are very relevant.

The aim of this work is to create an effective methodology of research of SSS of dentures with various surface reliefs and on the basis of studies to choose the most rational designs of dentures.

#### MATERIALS AND RESEARCH METHODS

During functioning, the human dentition apparatus experiences various loads and the displacements and deformations caused by them [5, 7].

The elements of orthopedic constructions are relatively thin plates and cylindrical shells operating under conditions of bending and plane tasks of the theory of elasticity. In the known calculation method of SSS of cylindrical folded shells [4], a variational method was applied to lower the dimensionality of the differential equations of bending and the plane problem. The disadvantages of this method include

the complex logic of forming a resolving system of equations and the need to solve differential equations for each structural element [6]. A known displacement method is based on the solutions of M. Levy (bending) and L. Fileon (plane problem) for rectangular plates [8], and also modifications of the displacement method and the mixed method [4, 6]. However, these methods are designed only for articulated support of the ends of the structure.

Therefore, for the study of orthopedic structures with a complex geometry, with gaps indisorders of the physical and mechanical properties of the material, under difficult boundary conditions, numerical methods based on discrete computational schemes are especially effective. FEM implies an explicit approximation of the solution on small subdomains – finite elements. Along with the FEM, another numerical method is actively developed – the BEM, where the basis is not finite-difference schemes, but the integral equation of the problem and its fundamental solutions [4, 6].

Based on the BEM relations, it is possible to construct the stiffness matrix of the elastic modulus. For plate systems, a transition from fundamental solutions to the FEM relations is also possible [8]. Therefore, studies aimed at the use of BEM to calculate the elements of orthopedic structures are very relevant.

A shell or slab-beam system, with which an orthopedic structure is modeled, is discretized into separate modules. The solution of the boundary value problem is performed with the boundary value of the variables of each module. Next, the initial parameters of all modules are determined, and the desired functions from the expressions [6].

$$\begin{aligned} DW(y) &= A_{11}DW(0) + A_{12}D\theta(0) - A_{13}M(0) - A_{14}Q(0) + B_{11}; \\ Z(y) &= -A_{11}N(0) - A_{12}S(0) + A_{14}EV(0) - A_{13}EU(0) + B_{s1} + P_1(y). \end{aligned} \quad (1)$$

Using the relations of the theory of elasticity, the functions of deflection  $w(x, y)$ , stresses  $\varphi(x, y)$ , and all other parameters of the elements of the folded shell are determined.

The order of alternation of the modules in the matrices is arbitrary, and the equations of equilibrium and compatibility of the movements of the nodes are composed in exactly the same way as for flat bar systems. The fundamental functions correspond to the case of articulated support, when  $r=s=n\pi/l_1$ .

The statics equations of shallow shells rectangular in terms of planes, deduced by Vlasov V.Z., have the form [6, 8]:

$$\begin{cases} DV^2\nabla^2 w(x, y) - \nabla_K^2 \varphi(x, y) = q_z(x, y); \\ \nabla^2 \nabla^2 \varphi(x, y) + Eh\nabla_K^2 w(x, y) = q_{xy}(x, y), \end{cases} \quad (2)$$

where  $w(x, y)$  – the transverse deflection;  $\varphi(x, y)$  – stress function;  $D$  – the cylindrical stiffness;  $E$  – Young's modulus;  $\nabla^2$  – Laplace operator;

The system of equations (2) is approximately solved by the Bubnov-Galerkin variational method under any boundary conditions and an arbitrary external load.

Equations (2) by substitution

$$w(x, y) = \nabla^2 \nabla^2 T(x, y) + \nabla_k^2 Z(x, y) \frac{1}{D}; \quad (3)$$

$$\varphi(x, y) = \nabla^2 \nabla^2 Z(x, y) - Eh \nabla_k^2 T(x, y)$$

come down to two separate equations with the same structure

$$\nabla^2 \nabla^2 \nabla^2 \nabla^2 T(x, y) + Eh \nabla_k^2 \nabla_k^2 T(x, y) \frac{1}{D} = q_z(x, y) \frac{1}{D}; \quad (4)$$

$$\nabla^2 \nabla^2 \nabla^2 \nabla^2 Z(x, y) + Eh \nabla_k^2 \nabla_k^2 Z(x, y) \frac{1}{D} = q_{xy}(x, y).$$

Expressions for generalized parameters of SSS shells are derived from the geometric and physical equations of the theory of shallow shells.

In order to determine the parameters of strength and stiffness depending on the shape of the fins of a complex curved surface of the denture, computer simulation of removable dentures was carried out. The calculation procedure was implemented using a finite element analysis program, and calculations were also performed using the BEM.

The denture design is a complex geometric surface of fragments of cylindrical outlines with a thickness of 2 mm.

We build the model using the SolidWorks (USA) version 2018 solid state parametric modeling package, which uses volumetric models as the main design objects.

We determine the strength of the plates under the action of a static load with an intensity of 200 kPa. Diagrams of deformations in the denture obtained as a result of calculation are presented in Fig. 1-2.

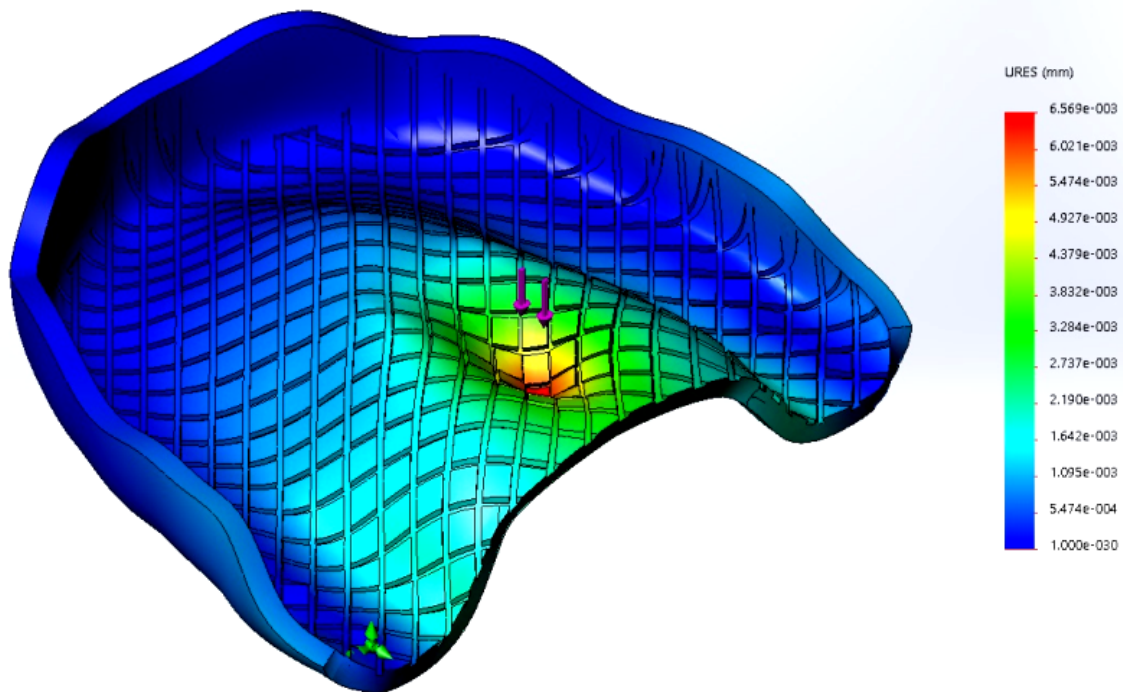


Fig. 1. Diagram of deformations in a denture with a rhombic lattice

Having performed the calculations using the boundary element method, we can compare them with the results of the FEM (Table).

Data analysis table shows that the results of two different methods are very close, which confirms the reliability of the calculations.

#### RESULTS AND DISCUSSION

The features of the proposed approach consist in modeling the stress state of dentures using two different methods, where close results are obtained,

which proves the reliability of both the simulation and the results. There are no restrictions on the application of the considered approaches, since both the FEM and the BEM provide a reliable solution to the problems posed.

The disadvantages of the study include the comparative complexity of constructing spatial models of dentures in the graphical editor of the FEM package and the formation of resolving systems of equations in the BEM.

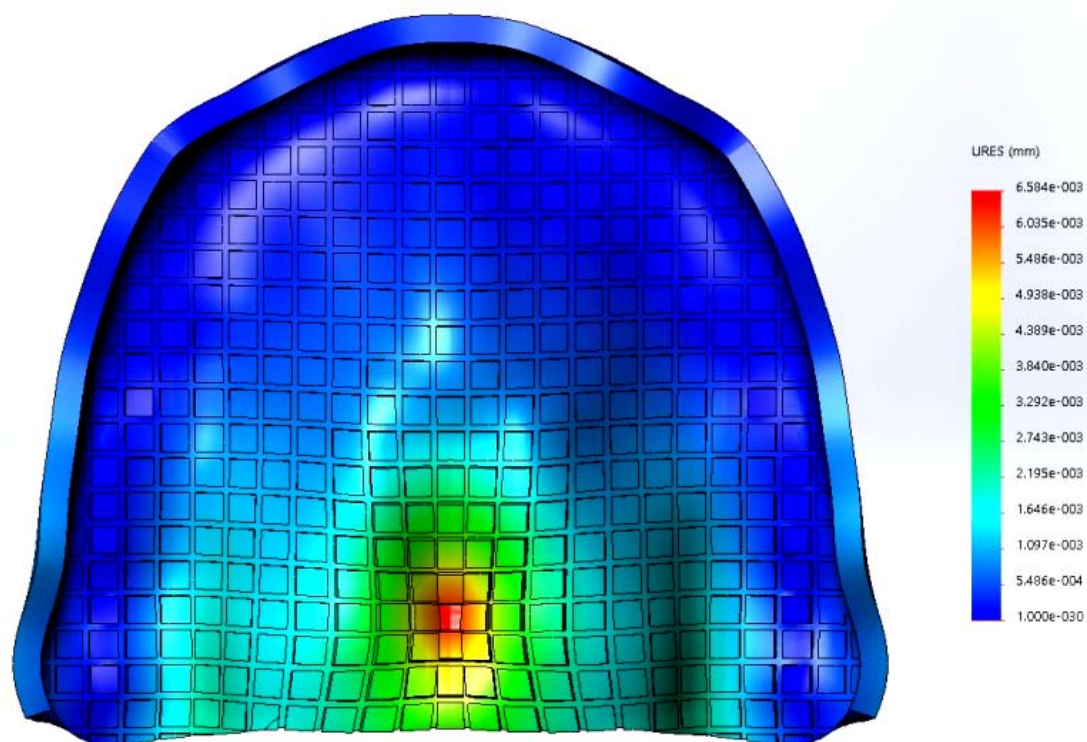


Fig. 2. Plot of vertical movements in a denture with a rectangular lattice

The lowest stress values occur in the denture with a rhombic lattice, which is by 5.9% less than a denture with a smooth surface, and by 18.78% less than a denture with a square lattice (strength characteristics are best for a denture with a rhombic lattice).

The equivalent displacements for a denture with a rhombic lattice are less by 3.864% than for a denture with a smooth surface, and by 8.52% less than for a denture with a square lattice.

### The calculated value of stresses and displacements in the denture

Smooth surface	
FEM	BEM
$\sigma=1,641 \text{ MPa}; \epsilon=3,581 \cdot 10^{-4} \Delta=6,263 \cdot 10^{-3} \text{ MM}$	$\sigma=1,6325 \text{ MPa} \epsilon=3,487 \cdot 10^{-4} \Delta=6,2496 \cdot 10^{-3} \text{ MM}$
Square lattice	
FEM	BEM
$\sigma=1,901 \text{ MPa}; \epsilon=2,771 \cdot 10^{-4} \Delta=6,582 \cdot 10^{-3} \text{ MM}$	$\sigma=1,8903 \text{ MPa}; \epsilon=2,7657 \cdot 10^{-4} \Delta=6,582 \cdot 10^{-3} \text{ MM}$
Rhombic lattice	
FEM	BEM
$\sigma=1,544 \text{ MPa}; \epsilon=2,625 \cdot 10^{-4} \Delta=6,021 \cdot 10^{-3} \text{ MM}$	$\sigma=1,5065 \text{ MPa}; \epsilon=2,5692 \cdot 10^{-4} \Delta=5,8927 \cdot 10^{-3} \text{ MM}$

Thus, we can conclude that a denture with a rhombic lattice on the surface has the best SSS rates in comparison with the other models considered (a denture with a smooth surface, a denture with a

square lattice). The practical value in the work is reduced to the possibility of modeling dentures on approximate objects in two different methods.

## CONCLUSIONS

1. The analysis of existing methods for modeling prototypes of dentures. It is shown that the most effective approaches for calculating the SSS of dentures are FEM and BEM.

2. The BEM algorithm and its calculated relations are presented as a result of integration of the differential equations of the statics of plates and shallow shells.

3. FEM is borrowed from SolidWorks professional package. At the same time, the creation of 3D models of dentures is shown in detail and clearly.

4. The calculations of the SSS of dentures according to the algorithms of the FEM and BEM. The calculation results are in good agreement with each other, which indicates the reliability of the created models and calculations.

5. From the results of the study it follows that a denture with a rhombic lattice on the surface has the best rates for SSS in comparison with other considered models.

Conflict of interests. The authors declare no conflict of interest.

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